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Starter Questions

Find the x-coordinates of the stationary points of the curve

$y = 5x^3 - 2x^2 - 3x + 10$

Determine the nature of the stationary points.
Determine for what values of x the curve is decreasing.

$\frac{}{dx} = 15x^2 - 4x - 3$ $\quad$ $\frac{}{dx} = 0$ *at a staionay point*

$$15x^2 - 4x - 3 = 0$$
$$(3x + 1)(5x - 3) = 0$$
$$x = -\frac{1}{3} \quad x = \frac{3}{5}$$

is a maximum.
is a minimum.
Hence the curve is decreasing for
.

H Integration

H1 Know and use the Fundamental Theorem of Calculus.

Students should be able to:

- understand that differentiation is the 'reverse' of integration and vice versa
- use $\int_a^b \mathrm{f}(x)\,\mathrm{d}x = \mathrm{F}(b) - \mathrm{F}(a)$

  where $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{F}(x)\right) = \mathrm{f}\left(x\right)$

  Note: the maximum level of difficulty for questions at AS requires students to use an integrand, $\mathrm{f}(x)$, where $\mathrm{f}(x)$ is the sum of terms of the form ax^n where n is rational and $n \neq -1$
- be able to find a function given its derivative and boundary condition.

H Integration

H2

Integrate x^n (excluding $n = -1$), and related sums, differences and constant multiples.

Integrate e^{kx}, $\frac{1}{x}$, sin kx, cos kx and related sums, differences and constant multiples.

Students should:

- know that:

$$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1} + c\,,\ n \neq -1$$

and

$$\int ax^n + bx^m \,\mathrm{d}x = a\int x^n \,\mathrm{d}x + b\int x^m \,\mathrm{d}x$$

- understand integration as the reverse of differentiation
- include a constant of integration when finding an indefinite integral.

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4.6 Integration

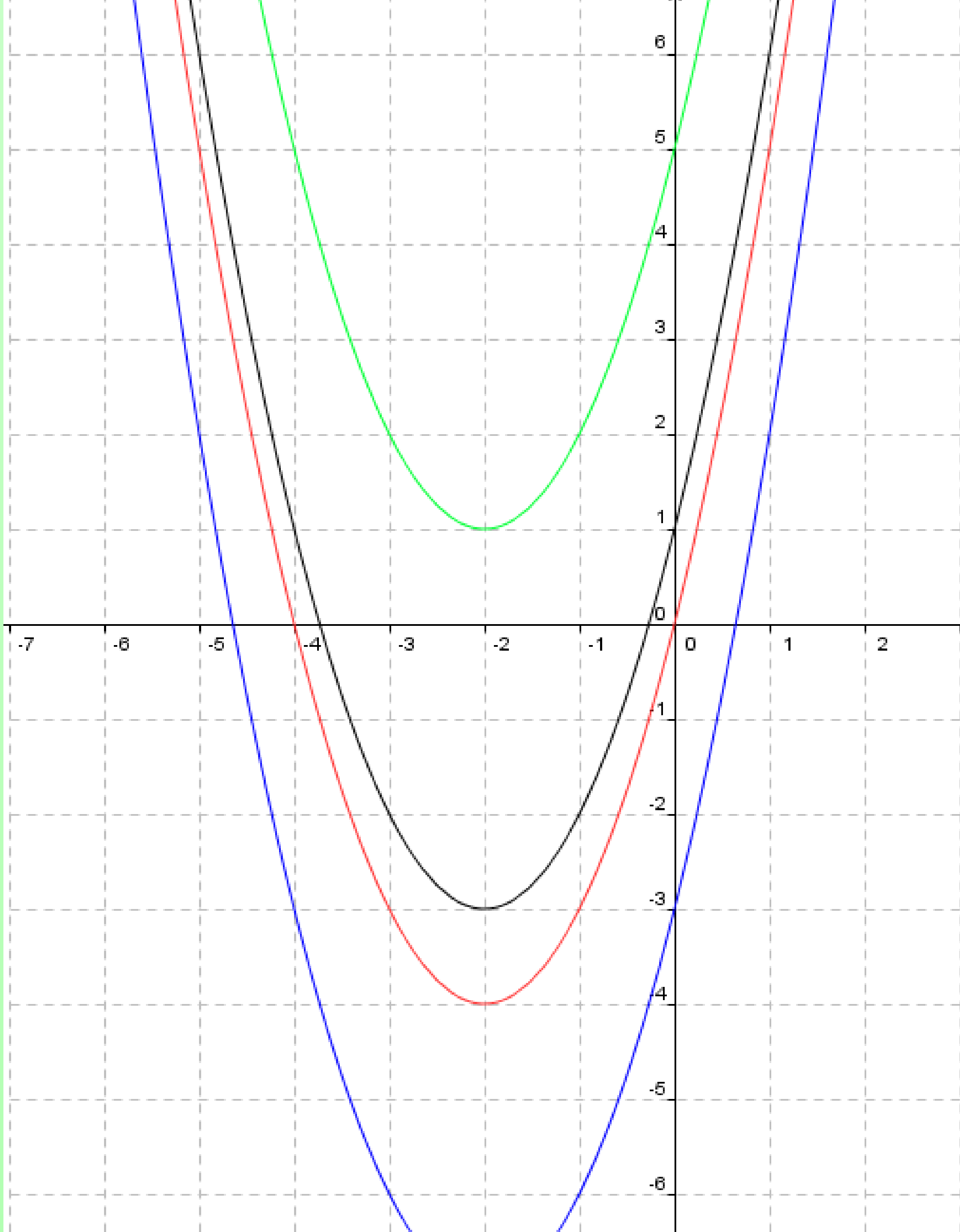
egration is the **reverse** of differentiation.

If we know $\frac{dy}{dx} = 2x + 4$

can we find the original equatio

int: Where did '2x' come from?

int: Where did '4' come from?



$= x^2+4x+5$

$y = x^2+4x+$

$y = x^2+4x$

$= x^2+4x - 3$

4.6 Integration

We can say that if... $\frac{dy}{dx}=2x+4$

...then

"general solution"

Or if

then

where is a constant called the "constant of integration"

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4.6 Integration

Notation

$$\int 2x + 4\, dx \;=\; x^2 + 4x + c$$

Integrate... ... with respect to x...

This is called INDEFINITE INTEGRATION because 'c' can have any value.

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4.6 Integration

general:

d 1 to the power and divide by the new powe

$$\boxed{\int ax^n \ dx = \frac{ax^{n+1}}{n+1} + \ c}$$

4.6 Integration

Example 1:

$$\int 3x^2\ dx$$

$$\int \frac{1}{3}x^5\ dx$$

$$\int 2x^4\ dx$$

¿ $\dfrac{2x^5}{5}+c$

$$\int 8\ dx$$

¿ $8x+c$

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4.6 Integration

Example 2:

$$\int \frac{1}{x^3}\,dx$$

$$\int \sqrt[3]{x^4}\,dx$$

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4.6 Integration

Example 3:

$$\int 4x^5 + 2x^2 - 3\,dx$$

$$\int 3x(x+5)^2\,dx$$

4.6 Integration

Example 4:

Find for

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4.6 Integration

Example 5:

Find for

Do exercise 13B

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4.6 Integration

$$\boxed{\int ax^n \; dx = \frac{ax^{n+1}}{n+1} + c}$$

Finding the constant of integration

We must be given further information if we are to find the value of this constant. For example, if we know the gradient function of a curve, and a point that it passes through, we can find the exact equation of the curve.

4.6 Integration

Example 6

Use the information given to find an expression for in terms of .

$$\frac{dy}{dx} = 3x^2 + 4x + 5$$ when

$$y = ¿ \int 3x^2 + 4x + 5 \, dx = x^3 + 2x^2 + 5x + c$$

Sub in : $10 = (1)3 + 2(1)2 + 5(1) + c$

$10 = 8 + c$

$2 = c$

$$\therefore y = x^3 + 2x^2 + 5x + 2$$

4.6 Integration

Example 7

The curve C with equation $y = f(x)$ passes through the point (4, 5). Given that $f'(x) = \frac{x^2 - 2}{\sqrt{x}}$, find the equation of C.

Sub in :

4.6 Integration

Example 8

The curve C with equation $y = f(x)$ passes through the point (4, 5). Given that $f'(x) = \frac{x^2 - 2}{\sqrt{x}}$, find the equation of C.

$$\therefore f(x) = \frac{2}{5}x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + \frac{1}{5}$$

4.6 Integration

You try:

The curve goes through the point and .
Find the equation of the curve.

The curve goes through the point and .
Find .

4.6 Integration

You try:

The curve goes through the point and .

Find the equation of the curve.

$$y=\int 2x^3\,dx\ ¿\ \frac{2x^4}{4}+c=\frac{1}{2}x^4+c$$

Sub in :

$$\therefore y=\frac{1}{2}x^4+8$$

4.6 Integration

You try:

The curve goes through the point and .
Find .

$$f(x)=\int 6x^2-6x\,dx$$

$$¿\frac{6x^3}{3}-\frac{6x^2}{2}+c=2x^3-3x^2+c$$

Sub in :

$$\therefore y=2x^3-3x^2+4$$